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Diffusion on random clusters and the parasite problem

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Abstract. In the parasite problem, a particle ('ant') diffuses randomly on a random percolation cluster in the limit of concentration → 0 ('lattice animal'). Monte Carlo simulations and scaling arguments show that for large animals the distance r travelled by this parasite increases as t^{1/z_A} with time t. We find $z_A \simeq 3.4$ on the simple cubic lattice and $z_A \simeq 2.6$ on the square lattice. This anomalous diffusion is in rough agreement with a generalisation of a suggestion by Alexander and Orbach. Heuristic arguments in favour of this suggestion are given. Also, we look at corrections to scaling for concentrations equal to the percolation threshold.

1. Introduction

The diffusion of a single particle on the random maze formed by a percolation process was dubbed 'ant in a labyrinth' by de Gennes (1976). Near the percolation threshold it has been studied extensively by scaling theories and Monte Carlo simulations (Mitescu *et al* 1978, Roussenq 1980, Vicsek 1981, Gefen *et al* 1983, Alexander and Orbach 1982, Ben-Avraham and Havlin 1982, Havlin *et al* 1983: see also Shender 1976), with a recent review by Mitescu and Roussenq (1983). In a generalisation of this problem, the labyrinth itself can also move and change its form (Kutner and Kehr 1983, see also Kehr 1983), with a speed slower than that of the diffusing ant. Or, instead of a random walk on a random structure one can look at self-avoiding walks on the same structure (Kremer 1981). Most of this work concerned averages over clusters of many different sizes, and was restricted to the region near the percolation threshold of the labyrinth. Gefen *et al* (1983) pointed out already that the results then depend on the type of average involved; and they looked at the behaviour for one fixed cluster size. They also pointed out that one should expect different diffusion properties on 'lattice animals', i.e. on clusters below the percolation threshold which are large compared with the correlation length.

The present paper continues these ideas by: (i) giving a scaling theory which also discusses in detail the animal limit (we call this special case of ants on an animal the parasite problem); (ii) presenting Monte Carlo results in two and three dimensions for parasites; and (iii) giving heuristic arguments to explain our parasite results. As a by-product we will obtain a derivation of the scaling law relating the percolation conductivity exponent μ to other critical exponents of percolation theory, like β or ν.

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2. Scaling theory

First we present a general scaling theory for diffusion on percolation clusters, both at the percolation threshold and for lattice animals. We assume the scaling form (Gefen *et al* 1983), with $\sigma = 1/\beta\delta = (1+1/\delta)/d\nu$:

$$r^2 = R_s^2 g(t/s^{z/D}, (p-p_c)s^{\sigma}) = t\xi^{-\theta} f_{\pm}(t/\xi^z, R_s/\xi) = t\xi^{-\theta}\phi_{\pm}(t/\xi^z, t/R_s^z), \tag{1}$$

where $z = 2+\theta = 2+(\mu-\beta)/\nu$ is the anomalous diffusion exponent (for concentrations p at the percolation threshold p_c the average end-to-end distance r walked by the ant is related to the time t, i.e. to the number of walk attempts, by $t \propto r^z$ asymptotically). Note that $r^2(t)$ is averaged only on clusters containing s sites and was denoted by Gefen *et al* (1983) as $\langle r^2(t)\rangle_s$. The average radius R_s of such clusters varies at $p = p_c$ as $s^{1/D}$, and the fractal dimensionality D in d dimensions is given by $D = d-\beta/\nu$ (Kirkpatrick 1978, Gefen *et al* 1981, Kapitulnik *et al* 1983a). The typical cluster radius is the correlation length $\xi \propto |p-p_c|^{-\nu}$. The fraction of sites belonging to the infinite network varies as $(p-p_c)^{\beta}$. All our scaling discussions refer to the limits $p \to p_c$, $s \to \infty$ and $t \to \infty$. Thus equation (1) is nothing but the percolation analogue of dynamical scaling (Hohenberg and Halperin 1977), applied to clusters of a given size s. We now discuss the three limits $(p-p_c)s^{\sigma} \to 0$ ('critical clusters'), $\to +\infty$ ('droplets'), and $\to -\infty$ ('animals'; very large clusters slightly below p_c seem to have the same asymptotic behaviour as medium sized clusters at $p = 0$. For according to Family and Coniglio (1980), the renormalisation group flows indicate that all clusters below p_c are 'animal'-like, provided $R_s \gg \xi$ (Stauffer 1979)). This discussion is the kinetic analogue of static results for the cluster radius R_s (Stauffer 1979, Essam 1980).

For this purpose we need also the assumption of 'strong' similarity, inherent in many discussions of the fractal nature of clusters (e.g. Gefen *et al* 1981, Kapitulnik *et al* 1983a, b, Stanley and Coniglio 1983). If we take a piece of a large cluster, with the piece centred about a cluster site and having the linear dimension L, then on the average this piece will be similar to a piece taken out of an even larger cluster, provided L is the same. (This assumption of strong similarity is not trivial since the density in the centre of very large clusters decreases with increasing cluster size (Herrmann 1979, Stauffer 1979). But here we require the centre to be a cluster site, as done e.g. by Forrest and Witten (1979) or Kapitulnik *et al* (1983*a*, *b*)).

2.1. Critical clusters

Equation (1) gives $r^2 = R_s^2 g(t/s^{z/D}, 0)$ at $p = p_c$. For small times t, s must cancel out according to the strong similarity hypothesis (consistent with our numerical data). Thus $g(y \to 0, 0) \propto y^{2/z}$ and $r^2 \propto t^{2/z}$. For long times, $g(y \to \infty, 0)$ approaches a constant, and $r^2 \propto R_s^2 \propto s^{2/D}$, since each site is visited with the same probability for very large times. Thus the ant practically measures the radius of gyration of the finite cluster (Gefen *et al* 1983, Essam 1980).

2.2. Droplets

For these large clusters above p_c equation (1) gives $r^2 \propto t\xi^{-\theta}\phi_+(t/\xi^z, t/R_s^z)$ and we have to distinguish three regimes for t. If $1 \ll t \ll \xi^z$, strong similarity requires r to be independent of s and of $p-p_c$, since no singularities are expected at fixed finite times t: $r^2 \propto t^{2/z}$ as for critical clusters. For $\xi^z \ll t \ll R_s^2\xi^{\theta}$ we have $r^2 \propto t\xi^{-\theta}\phi_+ \propto t(p-p_c)^{\mu-\beta}$

as is well known (de Gennes 1976). One may write this result as $t/t_\xi \propto (r/\xi)^2$ with $t_\xi \equiv \xi^z$ as the characteristic time of percolative diffusion. This result can be interpreted as describing usual diffusion with a step distance ξ, and a time step t_ξ. In the third regime, $t \gg R_s^2 \xi^\theta$ we have $r^2 \propto R_s^2$ again. (The distinction between $t < R_s^2 \xi^\theta$ and $t > R_s^2 \xi^\theta$ is made to give matching of normal diffusion and saturation regime, $r \propto t^{1/2}$ and $r =$ constant at the boundary $t = R_s^2 \xi^\theta$.)

2.3. Animals

Equation (1) gives $r^2 = t\xi^{-\theta} f_-(t/\xi^z, R_s/\xi)$. For short times, $1 \ll t \ll \xi^z$, $r^2 \propto t^{2/z}$ is again independent of s and $p - p_c$. For very large times we have again $r^2 \propto R_s^2$. For intermediate times we expect a different power behaviour, $t \propto r^{z_A} \equiv r^{2+\theta_A}$, with 'animal' exponents z_A and θ_A to be estimated below. (See also footnote (15) of Gefen *et al* 1983.) Strong similarity requires this law to be independent of s; therefore $r^2 = t\xi^{-\theta} f_-(t/\xi^z, \infty)$. Combination of both requirements gives $f_-(Y, \infty) \propto Y^{-1+2/z_A}$, and thus $r^2 \propto t^{2/z_A} \xi^{2(\theta_A - \theta)/z_A}$. Matching at the boundary shows that this result is expected to hold for $\xi^z \ll t \ll R_s^{z_A} \xi^{\theta - \theta_A}$. This result can be written in the more plausible form $t/t_\xi \ll (R_s/\xi)^{z_A}$, analogously to the droplet case at intermediate times.

Having discussed the leading behaviour in these three special cases, we now look at the correction terms for critical droplets, if p is very close but not equal to p_c. The distance r, depending on s and t as well as on p, should not feel the phase transition at $p = p_c$ if s and t are fixed. Therefore, if we write (1) as

$$r^2 = t^{2/z} \psi[t^{1/\nu z}(p - p_c), s^\sigma (p - p_c)] \tag{2}$$

then the function ψ should be analytic in $p - p_c$ and thus presumably in both of its arguments, if these arguments go to zero. Then the leading correction terms are

$$r^2 = t^{2/z}[c_0 + c_1(p - p_c)t^{1/\nu z} + c_2(p - p_c)s^\sigma + \ldots] \tag{3}$$

for the distance travelled by the ant in clusters of one fixed size s very close to the percolation threshold p_c. Usually one investigates the case where the ant starts running at a randomly selected lattice site (Mitescu and Roussenq 1983). In this case the averages, denoted as $\langle \ldots \rangle$, involve a sum over all cluster sizes s, with a weight equal to the probability $n_s s$ (Stauffer 1979, Essam 1980) that a site belongs to a cluster of size s. Summing over (3) we get an expansion of the same type:

$$\langle r^2 \rangle = t^{(2-\beta/\nu)/z} \Psi[t^{1/\nu z}(p - p_c)] = t^{(2-\beta/\nu)/z}(C_0 + C_1(p - p_c)t^{1/\nu z} + \ldots), \tag{4}$$

where the new scaling function Ψ is an integral involving the old scaling function ψ for the cluster size distribution. The modified power of t in the prefactor results from the averaging over all clusters (Gefen *et al* 1983).

3. Monte Carlo simulations

Our own Monte Carlo work concentrated on that exponent in the above scaling theories which our formulae do not predict from the critical exponents near the threshold, i.e. the animal exponent $z_A (= 2 + \theta_A)$. Within a single large lattice animal the ant, now called a parasite, diffuses according to the law $t \propto r^{z_A}$. Numerically, it is very difficult to study animals much larger than ξ near p_c. Thus instead we concentrate

on the case $p \to 0$ where ξ is small. Even then our analysis is hampered by the limited animal size.

We used the shape fluctuation algorithm (§ 2.3.3 of Stauffer 1979), where one produces various different configurations of one isolated cluster with s sites. This method allows, in contrast to all other Monte Carlo simulations of ants, to study diffusion in the limit $p = 0$; it is less efficient than other methods near p_c. Table 1 gives details of our computer runs for $p = 0$ only. Computer time limitations prevented us from simulating animals with more than 512 sites for $d = 3$ or 900 sites for $d = 2$. In three dimensions, memory limitations were also important though we stored the status of up to 30 lattice sites in one 60-bit word on the CDC Cyber 76 computer; somewhat larger clusters were studied at p_c, where their radius is smaller. At each step of the walk the parasite selects randomly a neighbour lattice site and attempts to jump to it. If that neighbour site is occupied, the ant jumps, otherwise it stays at its old place, as discussed by Vicsek (1981) or Mitescu and Roussenq (1983).

Table 1. Details of computer runs in the animal limit $p = 0$. s counts the number of sites in the animal, n_c the number of different clusters investigated for one size, n_a the number of ant walks for each cluster investigation, using configurations between t_1' and t_2' Monte Carlo steps per site in the cluster shape fluctuation algorithm. For three dimensions (upper part) five such runs were made for each of the three largest cluster sizes; for two dimensions, only one or two runs. A run for $s = 8^3$ took about 8000 s, the run for $s = 30^2$ about 1200 s on a CDC Cyber 76. More details are given by Wilke (1983).

s	n_c	n_a	t_1'	t_2'
2^3	34	80	1000	2000
3^3	34	270	1000	2000
4^3	68	640	200	1200
5^3	14	1250	200	395
6^3	82	2160	200	1215
7^3	60	3430	200	900
8^3	14	5120	200	395
20^2	14	5600	300	495
25^2	11	6875	300	465
30^2	11	9900	300	465

For three dimensions, figure 1 shows the average distance r travelled by the parasite at the time t, where r is measured in lattice constants and t in jump attempts. For the small clusters of figure 1(a), saturation is reached once r approaches the animal radius R_s. Thus only for large animals, like $s = 512$ in figure 1(b), can we try to look for an extended region where $1 \ll r \ll R_s$. Figure 2 shows (for $s = 6^3$, 7^3 and 8^3) the effective critical exponent $1/z_A = \mathrm{d}(\log r)/\mathrm{d}(\log t)$, determined by fitting tangents on the curves of figure 1. We see in all cases a nonlinear decrease of this effective exponent $1/z_A$ with time. For $s = 7^3$ an inflection point becomes visible at $1/z_A = 0.325 \pm 0.01$, which is more pronounced for $s = 8^3$ at $1/z_A = 0.315 \pm 0.005$. Since nothing is known to us on the asymptotic correction terms for large s we extrapolate linearly in $1/s$ to $1/z_A = 0.035$, or

$$z_A = 3.4 \pm 0.4, \qquad (d = 3), \tag{5}$$

for $s \to \infty$.

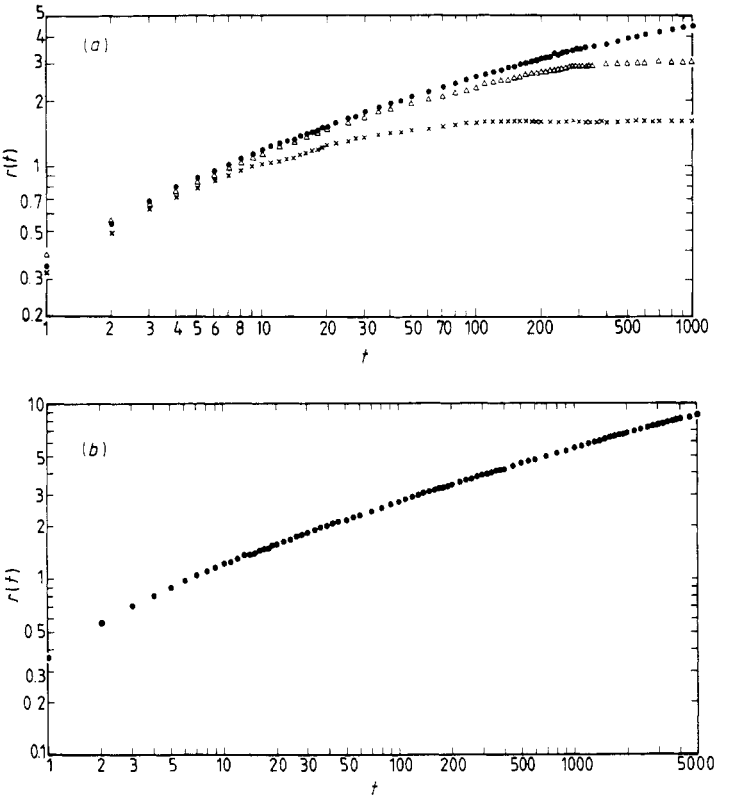

Figure 1. Variation of average end-to-end distance r of random walk of ant on lattice animal ('parasite') of a fixed size in the simple cubic lattice: (a, size = 2^3 (crosses), 3^3 (open triangles) and 4^3 (full circles); (b), size = 8^3.

Analogous determinations were made for $d = 3$ at $p = p_c = 0.3117$ for $s = 9^3$, giving $z = 3.4 \pm 0.1$ for this cluster size, and no reliable exponent for smaller sizes. The scaling prediction for asymptotic value $z = 2 + (\mu - \beta)/\nu$ is about 3.8 ● 0.2 (Derrida *et al* 1983, Mitescu and Greene as cited by Mitescu and Roussenq 1983, Alexander and Orbach 1982), suggesting that the asymptotic exponent is larger than the effective z found a finite s, in agreement with the above extrapolation for animals. For $p = 0.1$ we found $z = 3.3 \pm 0.15$ in a cluster with 8^3 sites, i.e. a value in between the animal and the critical cluster exponent. For $p = 0.2$ we failed to get a reliable estimate. We expect the true exponent to be animal-like for all p below p_c.

Figure 3 shows that the saturation limit for $r(t)$ is presumably reached exponentially, though our comparison of a decay as exp(-constant t) and exp(-constant $t^{1/2}$) favours only slightly the first choice. Assuming this simple-exponential decay for sufficiently large time in the average distance *for fixed cluster size s*, we can calculate the asymptotic behaviour of the *average* $\langle r^2 \rangle \equiv \Sigma_s s n_s \langle r^2(t) \rangle_s$ over all cluster sizes s, which is measured in most of the experiments reviewed by Mitescu and Roussenq (1983). Slightly below p_c, for $t \to \infty$, this average is dominated by animals with $R_s \propto \sqrt{s}$ ($d = 3$; Parisi and Sourlas 1981), and $\log n_s \propto -s$ (Stauffer 1979). Strong similarity no longer applies for the time dependence over distances $\gg \xi$. If we assume nevertheless the constant in exp(constant t/R_s^2) above to be independent of s, then for the average over all clusters

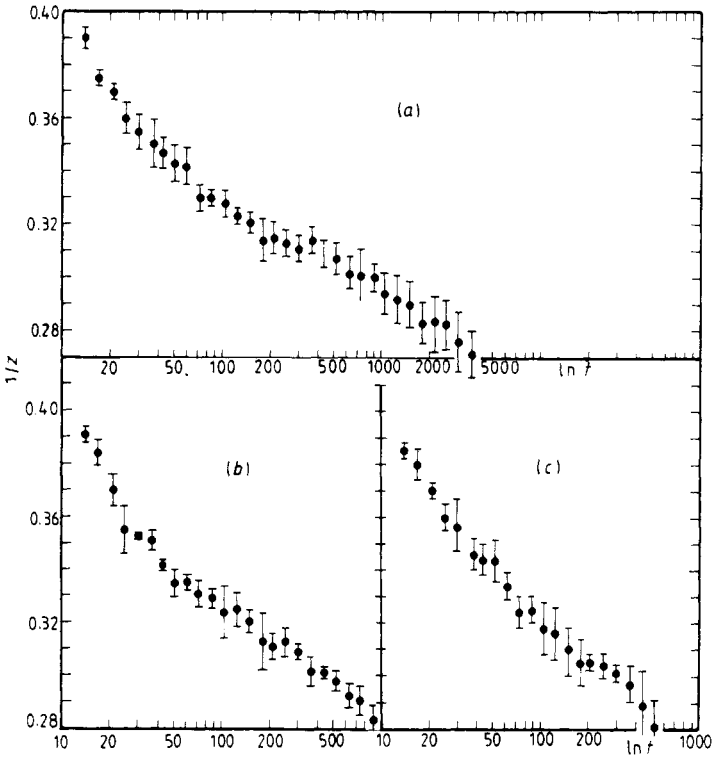

Figure 2. Variation of effective exponent z in $r \propto t^{1/z}$ for lattice animals of size 6^3, 7^3 and 8^3. (*a*), Cluster with 512 sites, (*b*) 393 sites and (c) 216 sites. For these larger clusters the error bar is the standard deviation from five runs, thus the probable error for each point is about half the error bar shown. ($1/z$ is calculated by fitting tangents to the curves of log r against log t.)

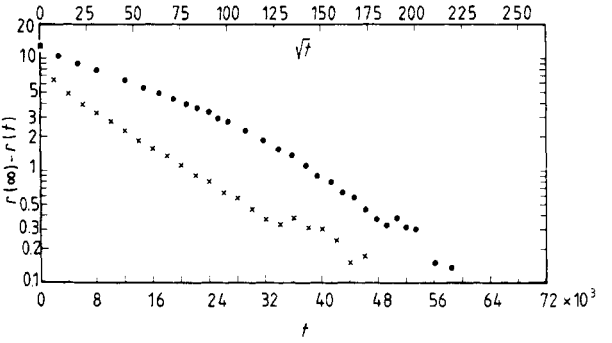

Figure 3. Approach to saturation for an animal with 512 sites. The crosses (t) follow a straight line better than the full circles ($\sqrt{t}$), suggesting that exp(-constant t) is a better approximation than exp(-constant $t^{1/2}$).

we get (see also Pandey *et al* 1983)

$$\log(\langle r(\infty)^2\rangle - \langle r(t)^2\rangle) \propto -t^{1/2}, \qquad (d=3, t\to\infty). \tag{6}$$

Even if we allow the constant in the exponential to decrease with increasing s we do

not find exp(-constant t) for the average over all s. This effect may explain why Mitescu and Roussenq needed many free parameters to fit the asymptotic behaviour of the average distance to a simple exponential decay.

Monte Carlo simulation on the square lattice gave similar results. The effective exponent was found to be $z_A = 2.65 \pm 0.03$ for animal size $s = 625$ and 2.63 ± 0.07 for $s = 900$. Extrapolation to $s \to \infty$ gives

$$z_A = 2.6 \pm 0.3, \qquad (d = 2). \tag{7}$$

4. Heuristic arguments and discussion

We now discuss a simple approximate model to explain at least roughly the parasite results (5) and (7). Figure 4 shows a simplified picture of the interior of a large critical cluster or animal. We assume a finite number of radial links, which are quasi-one-dimensional but not straight. At a distance r from the cluster centre they have a length $l = l(r) > r$. In addition to these radial links we have dangling bonds, which are also assumed to be quasi-one-dimensional, with an average length $l_d(r)$. Some of these dangling bonds, belonging to different radial links, may form parts of a circular link, as also shown schematically in our picture.

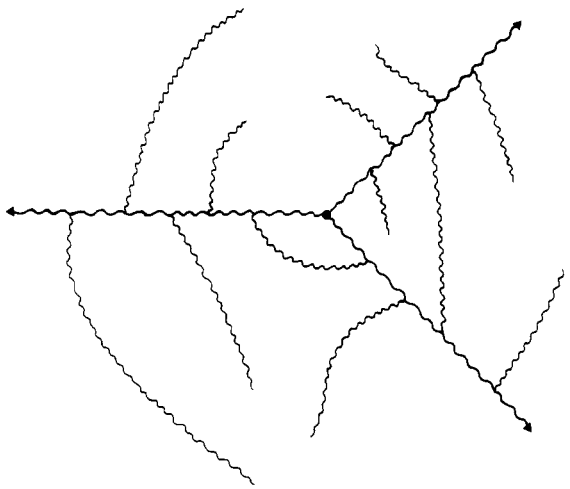

Figure 4. Schematic presentation of the region of a large animal or critical cluster near the origin of the ant walk. Radial links (ending with arrows) coexist with circular links and dead ends. This approximation may give the Alexander-Orbach scaling law for critical percolation clusters, and $z = 3$ for three-dimensional animals.

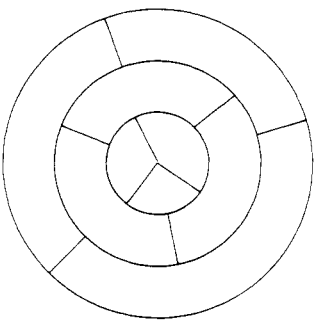

Figure 5. Schematic picture of a cluster consisting of spherical shells and radial links.

In the absence of dangling bonds we are left with some sort of a 'backbone' (Kirkpatrick 1978). Then the ant travels an end-to-end distance r in a time proportional to $l(r)^2$. With a dangling end attached to a finite fraction of sites on the radial links, this simple law $t \propto l^2$ is replaced by $t/t_d \propto l^2$. Here t_d is the average time the ant spends in a dangling end of length l_d before it continues to travel along the radial link. This

time t_d, which is related to the average return time to the origin, varies as l_d. The probability to cross a circular link is negligibly small compared with the probability to return to the origin of the circular link, since the crossing time varies as l_d^2 and not as l_d. Thus, the total time $t(r)$ to reach an end-to-end distance r varies as $l^2 l_d$ for the randomly walking ant in our picture.

How do the lengths l and l_d depend on the distance r? The total mass $M = M(r)$, measured as the number of sites, in that part of the cluster having a distance up to r from the origin, varies as r^D, where D is the same fractal or effective dimensionality as in equation (1) (Gefen *et al* 1981 or Kapitulnik *et al* 1983a give earlier references). Each radial link itself has the mass l and carries with it numerous dangling bonds of average mass l_d, originating from a finite fraction of the l sites on the radial link. Thus, the total mass of radial link and attached dangling bonds varies as $l + \text{constant}\ l_d l \propto l_d l$. Most of the mass is in the dangling bonds (Stanley and Coniglio 1983, Gefen 1983). Thus, $r^D \propto M \propto l_d l$ for large distances. When all the dangling bonds are eliminated, one is left with the 'backbone'. The backbone mass may be written as $l \propto r^{D_B}$, where $D_B = d - \beta_B/\nu$ is the backbone fractal dimensionality (Kirkpatrick 1978, Gefen *et al* 1981); thus $l_d \propto r^{D-D_B}$. Combining these two results, the total time to travel an end-to-end distance r becomes

$$t \propto l^2 l_d \propto r^{D+D_B}, \qquad \text{or } z = D + D_B \tag{8}$$

At $p = p_c$ for $d > 6$, one has $D = 4$ and $D_B = 2$ (Kapitulnik *et al* 1983b). Thus $D = 2D_B$, or $l_d \propto l$: the interior structure of a long dangling end is about the same as that of a radial link. If one accepts this relation for all dimensionalities and also for the animal limit, one ends up with

$$z = 3D/2. \tag{9}$$

For three-dimensional animals we have $D = 2$ exactly (Parisi and Sourlas 1981), and thus (9) gives $z_A = 3$, in rough agreement with (5). In two dimensions (9) yields $z_A = 2.34$, also not inconsistent with our equation (7). For eight dimensions, $z_A = 6$ since $D = 4$, if our result (9) would be exact.

Our geometrical model, figure 4, assumed a finite number of radial links. We now present another heuristic argument, relating the diffusion exponents to the geometrical structure of the infinite cluster and showing that, indeed, the number of radial links is small.

We consider a diffusion process starting at some site on a large cluster, and assume for simplicity that at a time t later all N sites within a sphere of radius $r(t)$ have equal probability to be occupied by the diffusion ant. Here $r(t) \propto t^{1/z}$ and $N \propto r^D$. At a time $t + \Delta t$, the only ants that can contribute to an increase in $r(t)$ are those which may propagate from the surface of the sphere. Their number varies as $C(r)/r^D$ where $C(r) \propto r^{-x}$ is the number of effective one-dimensional links connecting radially the sphere of radius r to the sphere of radius $r + \Delta r$, see figure 5. Each ant that diffuses from the r shell to the $(r + \Delta r)$ shell, increases the squared distance by $(r + \Delta r)^2 - r^2 \propto r$. Thus the diffusivity is

$$\mathrm{d}r^2(t)/\mathrm{d}t \propto rC(r)/r^D. \tag{10}$$

(For diffusion on a regular d-dimensional lattice, $D = d$ and $C(r) \propto r^{d-1}$. Hence $\mathrm{d}r^2/\mathrm{d}t = \text{constant}$, the well known Fickian result.)

Now we relate $C(r)$ to the conductivity of random resistor networks. Let us apply a voltage $\Delta V = 1$ between two shells at distances r_1 and $r_2 > r_1$ from the origin: $V(r_1) = 1$

and $V(r_2)=0$. Assuming an isotropic average conductivity $\propto r^{-\mu/\nu}$ we obtain a solution for the potential V:

$$V(r)=(r^{-y}-r_2^{-y})/(r_1^{-y}-r_2^{-y}), \qquad (r_1<r<r_2), \tag{11}$$

where $y=d-2-\mu/\nu$. The total current between the shells varies as $(r_1^{-y}-r_2^{-y})^{-1}$, and the total resistance $R=R(r_2)$ as $r_1^{-y}-r_2^{-y}$. Thus $\mathrm{d}R/\mathrm{d}r_2=r_2^{-y-1}$. On the other hand, $R(r_2+\Delta r)$ is obtained by adding in series to $R(r_2)$ the resistance of $C(r_2)$ parallel resistors of length Δr. Thus $r_2^{-y-1}\propto C(r_2)^{-1}$, and therefore

$$x=1-d+\mu/\nu. \tag{12}$$

This prediction gives $x=0.03$, ~ 0.2, and 1 for $d=2$, 3 and 6 respectively. Thus at low dimensions the number of effective intrashell connections is almost constant. (This $C(r)$ is not to be confused with the order of ramification, which is finite for critical clusters as well as for animals (Gefen *et al* 1981)). The fact that x is positive for high d is related to the amount of wiggle in the links: indeed there is a finite number of intershell links; however, each link contributes to the intershell resistance more than a unit resistance, and thus should be counted as *less* than one effective link. Note that (12) is consistent with scaling. From (10) and (12) we find for the average diffusivity

$$\mathrm{d}r^2/\mathrm{d}t\propto r^{-x-D+1}\propto r^{d-1-(\mu-\beta)/\nu+1}=r^{-\theta}, \tag{13}$$

as required (Gefen *et al* 1983). It would be nice to have direct Monte Carlo calculations of $C(r)$ to test this geometrical description of diffusion.

For critical clusters, (9) was already suggested on numerical grounds by Alexander and Orbach (1982), with some arguments in favour of (9) given by Rammal and Toulouse (1983). We find our argument more transparent, but not very reliable. Equation (9) may well turn out to be merely a good approximation, as the links-and-nodes picture of Skal, Shklovskii and de Gennes (Stanley and Coniglio 1983), or as Flory's approximation for self-avoiding walks. In particular, our Monte Carlo data for two-and-three-dimensional parasites are barely compatible with (9), and the picture itself becomes unreliable if D is smaller than 2 (as for $d=2$), where it would predict $l\propto l_{\mathrm{d}}\propto r^{D/2}<r$ in the approximation leading to (9). Moreover, the links and nodes picture breaks down at low dimensionalities (Gefen *et al* 1981) and should be replaced by another self-similar model.

It would be interesting to define a backbone dimensionality D_{B} for animals and to check our more general equation (8). These questions are of great interest since together with $z=2+(\mu-\beta)/\nu$ equations (8) and (9) link the conductivity exponent μ with other (geometrical) critical exponents of percolation theory. (It is obvious why (9) is invalid in one dimension: the above separation of radial links from dangling ends breaks down. On the other hand, (8) then yields the correct value $z=D+D_{\mathrm{B}}=2$ since backbone and cluster become identical.) For p above p_{c} we have a rather homogeneous, swiss-cheese like interior of very large droplets, and thus normal diffusion takes place: $z=2$ independent of dimension. These points should thus not be used as arguments against the Alexander–Orbach law.)

In summary, we presented (for both animals and critical percolation clusters) the first Monte Carlo data for ant diffusion as a function of cluster size. We offered an explanation, presumably approximate, for the animal exponents observed in two and three dimensions. The same explanation, applied to the percolation threshold, reproduces the scaling law of Alexander and Orbach and is an argument entirely different from that of Rammal and Toulouse.

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